

ment and testing of telescopic objectives, presented by the authors; F. W. Very, On the distribution of the Moon's Heat and its variation with the phase, presented by the author; Lick Observatory, Reports of observations of the total eclipse of the Sun, December 21-22, 1889, presented by the Observatory; T. W. Backhouse, The structure of the Sidereal Universe, presented by the author; L. Struve, Bestimmung des Mondhalbmessers, presented by the author; C. Flammarion, Copernic et la découverte du Système du Monde, presented by the author; W. Huggins, Address delivered at the Cardiff meeting of the British Association, presented by the author; Th. von Oppolzer, Traité de la détermination des orbites des comètes et des planètes, trad. par E. Pasquier, presented by Mr. R. E. S. Cooper; J. A. C. Oudemans, Die Triangulation von Java, Abth. III., presented by Professor Oudemans; the Photochronograph and its application to star transits, presented by Georgetown College Observatory; J. K. Rees, Catalogue of Prof. Rutherford's photographs, presented by Columbia College; Cincinnati Observatory, Charts and micrometrical measures of nebulae, presented by the Observatory; Lund Observatory, Observations de la zone $+ 35^{\circ}$ à $+ 40^{\circ}$, presented by the Observatory; Photographs of the Spectra of Solar prominences, presented by G. E. Hale; Photographs of the Cluster in *Hercules* and of *Jupiter*, taken at the Lick Observatory, presented by Professor Holden; Slater's Armillary Sphere, presented by Professor Slater; two drawings of *Jupiter*, presented by Mr. G. T. Gwilliam.

Note on the Lunar Theory. By Professor Cayley.

In the Lunar Theory, in whatever way worked out, the values ultimately obtained for the coordinates r, v, y should of course satisfy identically the equations of motion; and that they do so is the ultimate verification of the correctness of the results obtained. It can hardly be hoped for that such a verification will ever be made for Delaunay's results; and yet it would seem generally that the labour of such a verification of the results to any extent, while exceeding (and possibly greatly exceeding) that of obtaining these results by any method employed for that purpose, ought still to be, so to speak, a labour of the same order. And one can, moreover, imagine the process of verification so arranged as to be a process of mere routine which could be carried out by ordinary computers. But, however this may be, I think it is not without interest to exhibit the verification to a very small extent, viz. to e, m^4 .

I think there is an advantage in using capital letters for the arguments, and I accordingly write G (instead of Delaunay's g), to denote the mean anomaly.

The equations of motion are :

$$\begin{aligned}\frac{d^2r}{dt^2} - r \left\{ \cos^2 y \left(\frac{dv}{dt} \right)^2 \right\} + \frac{n^2 a^3}{r^2} &= \frac{d\Omega}{dr}, \\ \frac{d}{dt} \left\{ r^2 \cos^2 y \left(\frac{dv}{dt} \right) \right\} &= \frac{d\Omega}{dv}, \\ \frac{d}{dt} \left(r^2 \frac{dy}{dt} \right) + r^2 \sin y \cos y \left(\frac{dv}{dt} \right)^2 &= \frac{d\Omega}{dy}, \\ \Omega &= \frac{m' r^2}{r'^3} \left(\frac{3}{2} \cos^2 H - \frac{1}{2} \right), \text{ where } \cos H = \cos y \cos (v - v'),\end{aligned}$$

or say

$$\Omega = \frac{m^2 n^2 a'^3}{r'^3} r^2 \left\{ \frac{3}{2} \cos^2 y \cos^2 (v - v') - \frac{1}{2} \right\};$$

and thus the equations become

$$\begin{aligned}\frac{d^2r}{dt^2} - r \left\{ \cos^2 y \left(\frac{dv}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right\} + \frac{n^2 a^3}{r^2} \\ = \frac{m^2 n^2 a'^3}{r'^3} r \left\{ 3 \cos y \left\{ \frac{1}{2} + \frac{1}{2} \cos (2v - 2v') \right\} - \frac{1}{2} \right\}, \\ \frac{d}{dt} r^2 \cos^2 y \left(\frac{dv}{dt} \right)^2 = \frac{m^2 n^2 a'^3}{r'^3} r^2 \left(-\frac{3}{2} \cos^2 y \sin (2v - 2v') \right), \\ \frac{d}{dt} \left(r^2 \frac{dy}{dt} \right) + r^2 \sin y \cos y \left(\frac{dv}{dt} \right)^2 = \frac{m^2 n^2 a'^3}{r'^3} r^2 \left(-3 \sin y \cos y \frac{1}{2} + \frac{1}{2} \cos (2v - 2v') \right).\end{aligned}$$

To simplify as much as possible, take the Sun's orbit to be circular, i.e. $r' = a'$, $v' = mnt$; also neglect y^2 : the first and second equations are

$$\begin{aligned}\frac{d^2r}{dt^2} - r \left(\frac{dv}{dt} \right)^2 + \frac{n^2 a^3}{r^2} &= m^2 n^2 r \left\{ \frac{1}{2} + \frac{3}{2} \cos (2v - 2v') \right\} \\ \frac{d}{dt} r^2 \left(\frac{dv}{dt} \right) &= m^2 n^2 r^2 \left\{ -\frac{3}{2} \sin (2v - 2v') \right\};\end{aligned}$$

and if for convenience of working we write $a = 1$, $n = 1$, then the first equation may be written

$$\frac{1}{r} \frac{d^2r}{dt^2} - \left(\frac{dv}{dt} \right)^2 + \frac{1}{r^3} = m^2 \left\{ \frac{1}{2} + \frac{3}{2} \cos (2v - 2v') \right\},$$

and similarly the second equation is

$$\frac{1}{r^2} \frac{d}{dt} \left(r^2 \frac{dv}{dt} \right) = -\frac{3}{2} m^2 \sin (2v - 2v'),$$

and the third equation may be disregarded.

The two equations should be satisfied by Delaunay's values, putting therein $e' = 0$, $y = 0$; say by the values

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$$\begin{aligned} \frac{1}{r} = & 1 + \frac{1}{6}m^2 - \frac{179}{288}m^4 \\ & + \left(m^2 + \frac{19}{6}m^3 + \frac{131}{18}m^4\right) \cos 2D \\ & + \left(\frac{7}{8}m^4\right) \cos 4D \\ & + e \left(1 - \frac{7}{12}m^2\right) \cos G \\ & + e \left(\frac{33}{16}m^2\right) \cos 2D + G \\ & + e \left(\frac{15}{8}m + \frac{187}{32}m^2\right) \cos 2D - G; \end{aligned}$$

$$\begin{aligned} v = & t \\ & + \left(\frac{11}{8}m^2 + \frac{59}{12}m^3 + \frac{893}{72}m^4\right) \sin 2D \\ & + \left(\frac{201}{256}m^4\right) \sin 4D \\ & + 2e \sin G \\ & + e \left(\frac{17}{8}m^2\right) \sin (2D + G) \\ & + e \left(\frac{15}{4}m + \frac{263}{16}m^2\right) \sin (2D - G); \end{aligned}$$

Where

$$D = (1 - m)t, \quad G = \left(1 - \frac{3}{4}m^2\right)t, \quad (v' = mt).$$

The verification for the first equation is

	$\frac{1}{r} \frac{d^2r}{dt^2} =$	$-\left(\frac{dv}{dt}\right)^2 =$	$\frac{1}{r^3} =$	$m^2 \left(-\frac{1}{2} - \frac{3}{2}(\cos 2v - 2v')\right) =$	
Const.		-1	$+1$	$=0$	
			$+\frac{1}{2}m^2$	$-\frac{1}{2}m^2$	$=0$
	$+2m^4$	$-\frac{121}{32}m^4$	$-\frac{9}{32}m^4$	$-\frac{33}{16}m^4$	$=0$
Cos 2D	$+4m^2$	$-\frac{11}{2}m^2$	$+3m^2$	$-\frac{3}{2}m^2$	$=0$
	$+\frac{14}{3}m^3$	$-\frac{85}{6}m^3$	$+\frac{19}{2}m^3$		$=0$
	$+\frac{64}{9}m^4$	$-\frac{539}{18}m^4$	$+\frac{137}{6}m^4$		$=0$
Cos 4D	$+8m^4$	$-\frac{161}{16}m^4$	$+\frac{33}{8}m^4$	$-\frac{33}{16}m^4$	$=0$
Cos G	e	$-4e$	$3e$		$=0$
	$-\frac{9}{4}em^2$	$+em^2$	$-\frac{3}{4}em^2$		$=0$
Cos 2D + G	$e \frac{193}{16}m^2$	$e \left(-\frac{73}{4}m^2\right)$	$e \left(\frac{147}{16}m^2\right)$	$e(-3m^2)$	$=0$
Cos 2D - G	$em \left(\frac{15}{8}\right)$	$em \left(-\frac{15}{2}\right)$	$em \left(\frac{45}{8}\right)$		$=0$
	$em^2 \left(-\frac{5}{32}\right)$	$em^2 \left(-\frac{187}{8}\right)$	$em^2 \left(\frac{657}{32}\right)$	$em^2(3)$	$=0$

That the second equation is

$$\begin{array}{rclcl}
 \frac{2}{r} \frac{dr}{dt} \cdot \frac{dv}{dt} = & + \frac{d^2v}{dt^2} = & + \frac{3}{2} m^2 \sin(2v - 2v') = \\
 \text{Sin } 2D & + 4m^2 & - \frac{11}{2} m^2 & + \frac{3}{2} m^2 & = 0 \\
 & + \frac{26}{3} m^3 & - \frac{26}{3} m^3 & & = 0 \\
 & + \frac{142}{9} m^4 & - \frac{142}{9} m^4 & & = 0 \\
 \text{Sin } 4D & + \frac{21}{2} m^4 & - \frac{201}{16} m^4 & + \frac{33}{16} m^4 & = 0 \\
 \text{Sin } G & e \cdot 2 & + e \cdot -2 & & = 0 \\
 & e \cdot 3m^2 & + e \cdot 3m & & = 0 \\
 \text{Sin } 2D + G & e \cdot \frac{129}{8} m^2 & e \cdot -\frac{153}{8} m^2 & e \cdot 3m^2 & = 0 \\
 \text{Sin } 2D - G & e \cdot \frac{15}{4} m & e \cdot -\frac{15}{4} m & & = 0 \\
 & e \cdot \frac{71}{16} m^2 & e \cdot -\frac{23}{16} m^2 & e \cdot -3m^2 & = 0
 \end{array}$$

and the verification is thus completed.

The following intermediate results may be recorded ; $\frac{1}{r} = 1 + \chi$,

$$\begin{array}{rclcl}
 \chi^2 = & - \log r = & \frac{dv}{dt} = \\
 & \frac{19}{36} m^4 & \frac{1}{6} m^2 - \frac{255}{288} m^4 & 1 \\
 \text{Cos } 2D & + \frac{1}{3} m^4 & m^2 + \frac{19}{6} m^3 + \frac{64}{9} m^4 & \frac{11}{4} m^2 + \frac{85}{12} m^3 + \frac{539}{36} m^4 \\
 \text{Cos } 4D & + \frac{1}{2} m^4 & \frac{5}{8} m^4 & \frac{201}{64} m^4 \\
 \text{Cos } G & + \frac{1}{3} em^2 & e \left(1 - \frac{3}{4} m^2 \right) & e \left(2 - \frac{3}{2} m^2 \right) \\
 \text{Cos } 2D + G & + em^2 & e \left(\frac{25}{16} m^2 \right) & e \left(\frac{51}{8} m^2 \right) \\
 \text{Cos } 2D - G & + em^2 & e \left(\frac{15}{8} m + \frac{171}{32} m^2 \right) & e \left(\frac{15}{4} m + \frac{143}{16} m^2 \right)
 \end{array}$$

$$\frac{1}{r} \frac{dr}{dt} =$$

$$\begin{array}{rcl}
 \text{Sin } 2D & 2m^3 + \frac{13}{3} m^3 + \frac{71}{9} m^4 \\
 \text{Sin } 4D & \frac{5}{2} m^4 \\
 \text{Sin } G & e \left(1 - \frac{3}{2} m^2 \right) \\
 \text{Sin } 2D + G & e \left(\frac{75}{16} m^2 \right) \\
 \text{Sin } 2D - G & e \left(\frac{15}{8} m + \frac{51}{32} m^2 \right)
 \end{array}$$

These were in fact made use of for finding the foregoing values of r , $\frac{d^2r}{dt^2}$, &c.

Colour Changes in the Markings on the Surface of the Planet Jupiter.
By E. E. Barnard.

During the twelve years that I have observed *Jupiter* I have often been struck with the decided changes of colour in the different markings on his surface. A careful study of the numerous details during this time has led to the discovery that the red colour of any of the markings is an indication of their age; or, in other words, when a spot or marking (other than the white spots) first appears it is dark or black, but after some time turns red. I have repeatedly predicted to myself this transition upon the appearance of a dark spot, and have yet to find the first instance of a deviation from this striking rule.

As an illustration, I would mention a few cases in my own experience. The remarkable black spots of 1880 which appeared on the northern hemisphere of the planet in October and November of that year were at first black, and then terminated their career by forming a red belt around *Jupiter* (see *Publ. A. S. P.* No. 5).

The small inky-black spots which appeared on the northern edge of the equatorial belt in 1890 soon turned red, and are now almost the reddest objects on the planet.

The new red spot, which lies on a parallel with the southern edge of the great red spot, and follows it by about 140° , when first seen by me in the early part of August 1890, was dark, with no trace of red. It is now the most conspicuous object on the planet, and is of a deep red colour.

There is another marking similar to this in the same latitude, and preceding the great red spot by an hour or so. When first seen this was dusky, with no trace of red. On August 14 last I wrote in my note-book: "This dusky oblong marking has no trace of red now; perhaps it will turn red." It is now of a strong red colour.

I have noted this transformation in a number of other cases. It would, therefore, appear certain that as a rule markings of this class first appear dark, and later turn red.

I have been curious to know if the great red spot in its early history was also governed by this rule. When I first saw it in 1879 it was a deep red, and has been so ever since, except where partially obscured by white clouds, during the past six or seven years. I have fortunately found two early observations of this spot in 1872. These fully prove that the great red spot is no exception to the rule. In the *Observatory* for 1882, vol. v. p. 21, Mr. H. Corder says, in speaking of his observations of this object in 1872, March 13: "Its colour was not noted as red." In the same volume, p. 55, M. Terby confirms Mr. Corder's statement about the absence of colour in the spot. He has an observation 1872, January 28, and, referring to this, says: "I ought